AIAA JOURNAL
Vol. 35, No. 6, June 1997

New Scattering Kernel for Gas–Surface Interaction

C. Cercignani* and M. Lampis*
Politecnico di Milano, Milan 20133, Italy

In the frame of the kinetic theory of gases, some scattering kernels for gas–surface interaction are reviewed, and a new scattering kernel is introduced. Comparisons with experimental data for the drag coefficient in hypersonic free-molecular flow are made. The data obtained by the Göttingen group by means of rarefied gas wind tunnel experiments, at stagnation temperatures of the gas of the same order of the target temperature, are examined. Several other data regarding satellite conditions of flight are also considered.

Nomenclature

a, b, h	= parameters of the scattering kernels
C_D	= drag coefficient
C_H	= heat coefficient
C_L	= lift coefficient
C_n	= normal momentum coefficient
C_τ	= tangential momentum coefficient
$\mathbf{c}$	= molecular velocity
E	= energy of a molecule
f	= one-particle velocity distribution function
f_0	= Maxwellian distribution function in equilibrium with the wall
I_0	= modified Bessel function of zero order
k	= Boltzmann constant
m	= mass of molecule
$\mathbf{n}$	= unit vector normal to the wall
n_∞	= numerical density of incident beam
p_n, p_τ	= normal momentum and tangential momentum
R	= gas constant, k/m
$\mathcal{R}$	= scattering kernel
R_F	= reflection coefficient
r	= T_w/T_0
S	= speed ratio
T_w	= wall temperature
T_0	= stagnation temperature
T_∞	= temperature of incident beam
v_∞	= mean velocity of the incident beam
$\alpha_{NM}, \alpha'_{NM}$	= Knuth's accommodation coefficients
α_k	= Kuščer's accommodation coefficients
δ	= Dirac function
ρ	= density
σ	= parameter in Maxwell's model
$\sigma_\tau, \sigma', \sigma_E$	= Schaaf's accommodation coefficients
σ_t, σ_n	= parameters of the Cercignani–Lampis model

Subscripts

i	= incident
n or x	= normal to the wall
r	= reflected
t	= tangential to the wall
w	= wall
∞	= incident beam

Introduction

THE accuracy of the prediction of the aerodynamic coefficients in free-molecular and transition regimes is based on the use of sensible models of the gas–surface interaction process. A completely satisfactory theory of the atom–surface scattering should be based on a quantum mechanical treatment of the inelastic scattering. However, the classic theory may be used correctly and with less effort in some problems involving large atom–surface energy and momentum transfers, as in the case of the drag forces on artificial Earth satellites.

Simplified models of the scattering of gas molecules from a solid surface are currently used in kinetic theory. The first model is due to Maxwell.[1] It is assumed that the reemitted molecules are partly reflected by the wall in a specular fashion and partly diffused in a completely accommodated fashion, according to a Maxwellian distribution corresponding to the wall temperature. Maxwell's model assumes that every momentum Φ_r of the distribution function of the reemitted molecules is given by $\Phi_r = (1 - \sigma)\Phi_i + \sigma\Phi_w$, where Φ_i is the momentum of the impinging molecules, Φ_w is the momentum of the molecules reemitted according to thermal diffusion, and σ is the accommodation coefficient.

Three different accommodation coefficients were introduced by Schaaf[2]:

$$\sigma' = \frac{p_{ni} - p_{nr}}{p_{ni} - p_{nw}}, \qquad \sigma_\tau = \frac{p_{\tau i} - p_{\tau r}}{p_{ni} - p_{nw}}, \qquad \sigma_E = \frac{E_i - E_r}{E_i - E_r} \quad (1)$$

where p_{ni}, $p_{\tau i}$, and E_i are normal momentum, tangential momentum, and energy, respectively, brought to the unit area of the surface by the incident molecules; p_{nr}, $p_{\tau r}$, and E_r are the same quantities carried away by those molecules after reemission from the surface; and p_{nw}, $p_{\tau w}$, and E_w are the reemitted quantities according to thermal diffusion.

It has been remarked that the definition of the accommodation coefficient for normal momentum is unsatisfactory for applications in which p_{in} is approximately equal to p_{nw} because the denominator vanishes.[3,4] To avoid this singularity, Knuth[3] and Liu et al.[4] used vector quantities and arrived at the definition of two new accommodation coefficients:

$$\alpha_{NM} = \frac{p_{ni} + p_{nr}}{p_{ni} + p_{nw}}, \qquad \alpha'_{NM} = \frac{p_{ni} + p_{nr}}{p_{ni}} \quad (2)$$

In the early 1960s, Nocilla[5] introduced a boundary condition for the Boltzmann equation, according to which the distribution function of the molecules reemitted by the wall is a "shifted Maxwellian," containing three adjustable parameters. The model was supplemented by Hurlbut and Sherman[6] to provide a connection between the exit distributions and the incident flow [Hurlbuth, Sherman, and Nocilla (HSN) model]. In the following section, we come back on this argument.

In the late 1960s, the problem of writing boundary conditions for the Boltzmann equation in a rigorous manner was formulated in terms of the so-called scattering-kernel theory.[7–10] Under some assumptions, the distribution functions of the reemitted and of the impinging molecules are related by

$$c_n f(\mathbf{c}) = \int_{c'_n < 0} |c'_n| f(\mathbf{c}') \mathcal{R}(\mathbf{c}' \to \mathbf{c})\, d\mathbf{c}' \qquad (c_n \geq 0) \quad (3)$$

Received Aug. 5, 1995; revision received Feb. 3, 1997; accepted for publication Feb. 3, 1997.

*Professor of Theoretical Mechanics, Dipartimento di Matematica, Piazza Leonardo da Vinci.

where $\boldsymbol{n}$ is the unit vector normal to the wall pointing into the gas. The kernel $\mathcal{R}(\boldsymbol{c}' \to \boldsymbol{c})$, which is the probability density that a molecule impinging on the wall with velocity $\boldsymbol{c}'$ is reemitted at practically the same position and the same time with velocity $\boldsymbol{c}$, gives a simplified model of gas–surface interaction. $\mathcal{R}(\boldsymbol{c}' \to \boldsymbol{c})$ satisfies the well-known properties of positivity, normalization, and reciprocity[2–4]:

$$\mathcal{R}(\boldsymbol{c}' \to \boldsymbol{c}) \geq 0 \tag{4a}$$

$$\int_{c_n > 0} \mathcal{R}(\boldsymbol{c}' \to \boldsymbol{c})\, \mathrm{d}\boldsymbol{c} = 1 \tag{4b}$$

$$|c_n'| f_0(\boldsymbol{c}') \mathcal{R}(\boldsymbol{c}' \to \boldsymbol{c}) = |c_n| f_0(\boldsymbol{c}) \mathcal{R}(-\boldsymbol{c} \to -\boldsymbol{c}') \tag{4c}$$

where f_0 is a Maxwellian distribution in equilibrium with the wall.

At the same time, researchers engaged in solid-state physics also were studying the scattering of atoms by solid surfaces, but according to a different point of view. For instance, a nearly monoenergetic atomic beam hitting a solid surface is considered and an intensity R_F is defined for each of the outgoing beams as the ratio of the flux of atoms in the diffracted beam F over the incident flux of atoms.[11] R_F must satisfy the relation

$$\sum_F R_F = 1 \tag{5}$$

In a subsequent paper,[12] R_F is called the reflection coefficient. We remark here that this reflection coefficient essentially may be identified with the scattering kernel $\mathcal{R}(\boldsymbol{c}' \to \boldsymbol{c})$.

Some Models of Scattering Kernels

In the frame of the scattering-kernel theory, Maxwell's model is equivalent to choosing

$$\mathcal{R}(\boldsymbol{c}' \to \boldsymbol{c}) = (1 - \sigma)\delta(\boldsymbol{c} - \boldsymbol{c}_R') + \sigma f_0(\boldsymbol{c}) |\boldsymbol{c} \cdot \boldsymbol{n}|$$
$$(\boldsymbol{c} \cdot \boldsymbol{n} > 0;\ \boldsymbol{c}' \cdot \boldsymbol{n} \leq 0) \tag{6}$$

where $\boldsymbol{c}_R' = \boldsymbol{c}' - 2\boldsymbol{n}[\boldsymbol{n} \cdot \boldsymbol{c}']\boldsymbol{c}'$ and σ is the fraction of molecules diffusely scattered; f_0 is given by

$$f_0 = \left[2\pi(RT_w)^2\right]^{-1} \exp\left[-c^2(2RT_w)^{-1}\right] \tag{7}$$

the normalization being chosen in such a way that Eq. (4b) is satisfied.

Another kernel with two adjustable parameters [Cercignani–Lampis (CL) model][13] was proposed by the authors of the present paper:

$$\mathcal{R}(\boldsymbol{c}' \to \boldsymbol{c}) = \frac{[\sigma_n \sigma_t (2 - \sigma_t)]^{-1}}{2\pi(RT_w)^2} c_n$$
$$\times \exp\left\{ -\frac{c_n^2 + (1 - \sigma_n)c_n'^2}{2RT_w \sigma_n} - \frac{[\boldsymbol{c}_t - (1 - \sigma_t)\boldsymbol{c}_t']^2}{\sigma_t(2 - \sigma_t) 2RT_w} \right\}$$
$$\times I_0\left(\frac{\sqrt{1 - \sigma_n}\, c_n c_n'}{\sigma_n RT_w} \right)$$
$$(c_n' < 0;\ c_n > 0;\ 0 \leq \sigma_t \leq 2;\ 0 \leq \sigma_n \leq 1) \tag{8}$$

In Eq. (8) the two parameters σ_n and σ_t depend on the physical nature of the gas and the wall as well as on the temperature of the latter: σ_n and σ_t are the accommodation coefficients for normal energy and for tangential momentum, respectively. The accommodation coefficient for tangential energy turns out to be $\sigma_t(2 - \sigma_t)$. Model (8) recovers, as limiting cases, the specular reemission (for $\sigma_n = \sigma_t = 0$) and the diffuse reemission (for $\sigma_n = \sigma_t = 1$). The kernel (8) also was obtained[14] by solving a Fokker–Planck equation, assumed as a model for the dynamics of a gas molecule captured by the wall.

When the CL model was presented, many results were already known from experiments in which a nearly monochromatic beam of molecules impinges on a wall, and the emerging molecules are counted. The reemission patterns in the plane of incidence, calculated by employing the above kernel, turned out to have a lobular shape[13,15] and, if compared with experimental data, appeared to be much more satisfactory than the patterns given by Maxwell's model.

The CL model also was applied to the calculation of drag, lift, and heat transfer coefficients in free-molecular flow,[16–18] but no comparison with experimental data was made at that time. For the definitions of C_D, C_L, C_H used in the present paper, see the Appendix.

After many years, experimental data about these coefficients in hypersonic free-molecular flow were obtained by the research group at Göttingen[19] and compared with the theoretical results given by the CL model. The analysis of the experimental data seems to lead to the conclusion that the accommodation coefficients for normal energy and for tangential energy are equal,[20] so that a relationship between the two parameters of the CL model should be introduced a posteriori [$\sigma_n = \sigma_t(2 - \sigma_t)$], although the two parameters were a priori independent of each other. This result, if confirmed by other sets of data, introduces a limitation of the flexibility of the model, which becomes a one-parameter model. Other interesting aspects of this comparison are shown in the figures appearing in Ref. 19, on which some remarks were made in Ref. 20.

Recently Proposed Model of Scattering Kernel: CLL Model

The Nocilla model was the subject of research not only in the 1960s but even very recently.[21–26] Hurlbut[23] presented a method for coordinating the classic transfer coefficient description with a parametric exit velocity distribution given by the HSN model. Actually, observations from molecular beam studies using high-speed incident flow, and molecular dynamic simulation, show that the exit velocity in any particular direction may be represented best by drifting Maxwellian distribution function.[23–27] Nocilla's model, however, is not formulated in the frame of the scattering-kernel theory, so that it cannot be used in connection with some methods of solution of the Boltzmann equation in the transition regime, as, for instance, the variational method,[7–9] which requires boundary conditions formulated by means of a scattering kernel. Some applications of the variational method to the calculation of velocity slip and temperature jump coefficients, for monatomic and polyatomic gases and for mixtures, in which use is made of boundary conditions for the Boltzmann equation formulated according to the scattering-kernel theory, were presented at rarefied gas dynamics symposia.[28–31] It seems interesting then to try to formulate a new scattering kernel that takes into account the remark that a shifted Maxwellian is a good model of a reemitted distribution function.

An attempt to exploit the main idea contained in Nocilla's model, without renouncing the scattering-kernel formulation, was proposed recently.[32–34] In those papers, the technique employed to construct a new kernel is a purely mathematical method already proposed[5] but never used. This approach is useful for reformulating rigorously, in the frame of the scheme of the scattering-kernel theory, reemission models that approximate satisfactorily the experimental results but do not satisfy the fundamental properties of the scattering kernels. We recall here briefly this kernel as well. Cercignani[32] introduced the shifted Mawellian distribution function

$$\mathcal{R}(\boldsymbol{c}' \to \boldsymbol{c}) = (2/\pi)\beta_w^2 b(1 - a^2)^{-2} c_x$$
$$\times \exp\left[-\beta_w (1 - a^2)^{-1} |\boldsymbol{c} - a\boldsymbol{c}_R'|^2\right] \tag{9}$$

containing the two parameters $-1 < a < 1$, $b > 0$, while $\beta_w = (2RT_w)^{-1}$. In Eq. (9), the direction of the outward normal to the wall is assumed as the x axis. The introduction of this tentative kernel was suggested by Nocilla's boundary condition. Because the kernel (9) does not satisfy the normalization and reciprocity properties, a suitable correction term has been added[32] that produces a scattering kernel obeying relations (4a) and (4b):

$$\mathcal{R}(\boldsymbol{c}' \to \boldsymbol{c}) = \mathcal{R}(\boldsymbol{c}' \to \boldsymbol{c}) + c_x f_0(\boldsymbol{c})[1 - H(-\boldsymbol{c}')][1 - H(\boldsymbol{c})]/I \tag{10}$$

where $H(\mathbf{c}')$ and I are obtained from

$$H(\mathbf{c}') = \int_{c_x > 0} \mathcal{R}(-\mathbf{c}' \to \mathbf{c})\, \mathrm{d}\mathbf{c} \tag{11}$$

$$I = \int_{c_x > 0} c_x f_0(\mathbf{c})[1 - H(\mathbf{c})]\, \mathrm{d}\mathbf{c} \tag{12}$$

We remark that Eq. (10) can be rewritten as

$$\mathcal{R}(\mathbf{c}' \to \mathbf{c}) = H(-\mathbf{c}')S_1(\mathbf{c}' \to \mathbf{c}) + [1 - H(-\mathbf{c}')]S_2(\mathbf{c}' \to \mathbf{c}) \tag{13}$$

where S_1 and S_2 are two normalized kernels and $H(\mathbf{c}')$ is a sort of generalized accommodation coefficient depending on the incoming velocity:

$$S_1(\mathbf{c}' \to \mathbf{c}) = \frac{\mathcal{R}(\mathbf{c}' \to \mathbf{c})}{H(-\mathbf{c}')}$$
$$S_2(\mathbf{c}' \to \mathbf{c}) = \frac{c_x f_0(\mathbf{c})[1 - H(\mathbf{c})]}{I} \tag{14}$$

The parameter b, appearing as a factor in expression (9), represents the weight of $\mathcal{R}(\mathbf{c}' \to \mathbf{c})$ in Eq. (10), i.e., the weight of the first kernel $S_1(\mathbf{c}' \to \mathbf{c})$ in Eq. (13); b has some similarity with the parameter σ appearing in Maxwell's model. The parameter a expresses the assumption that, in general, the temperature T_s of the shifted Maxwellian is different from the temperature T_w of the surface, according to $T_s = (1 - a^2)T_w$. Moreover, in the argument of the Maxwellian, the vector $a\mathbf{c}'_R$ has the same direction as $\mathbf{c}'_R$, but its magnitude is reduced. As a consequence, $\mathcal{R}(\mathbf{c}' \to \mathbf{c})$ satisfies the reciprocity relation (4c).

According to kernel (9), the function $H(\mathbf{c}')$ is limited only for negative values of a, so that only for $a < 0$, with a convenient choice of b, $H(\mathbf{c}') < 1$, as it must be. To allow positive values of a and to extend the validity of the model, in Refs. 33 and 34 a third parameter, $h > 0$, was introduced and the following kernel [Cercignani, Lampis, and Lentati (CLL) model] was proposed:

$$\mathcal{R}(\mathbf{c}' \to \mathbf{c}) = (2/\pi)\beta_w^2 b(1 + h)(1 - a^2)^{-2} c_x \times \exp\left[-\beta_w h(1 - a^2)^{-1}\left(c_x^2 + c_x'^2\right) - \beta_w(1 - a^2)^{-1}\left|\mathbf{c} - a\mathbf{c}'_R\right|^2\right] \tag{15}$$

After some calculations we obtain

$$H(\mathbf{c}') = H_0(c'_x) + a\beta_w^{\frac{1}{2}} c'_x H_1(c'_x)$$
$$H_0(c'_x) = b \exp\left[-\beta_w(h + a^2)(1 - a^2)^{-1} c_x'^2\right]$$
$$H_1(c'_x) = \pi^{\frac{1}{2}} b(1 - a^2)^{-\frac{1}{2}}(1 + h)^{-\frac{1}{2}} \times \exp\left[-\beta_w h(1 + a^2 + h)(1 + h)^{-1}(1 - a^2)^{-1} c_x'^2\right] \times \left\{1 + \operatorname{erf}\left[a\beta_w^{\frac{1}{2}}(1 - a^2)^{-\frac{1}{2}}(1 + h)^{-\frac{1}{2}} c'_x\right]\right\} \tag{16}$$

$$I = 1 - (K_{01} + aK_{11}) \tag{17}$$

where

$$K_{01} = b\frac{1 - a^2}{1 + h}, \qquad K_{11} = \pi^{\frac{1}{2}} b \frac{(1 + h)(1 - a^2)}{[(1 + h)^2 - a^2]^{\frac{3}{2}}} \tag{18}$$

The parameters a, b, and h must be linked in such a way that $I > 0$ and $H(c_x) \leq 1$. An approximate estimate of the maximum value of b for any a and h was given previously[33,34] as

$$b \leq b_{\max} = \left\{1 + a\left[2\pi(eh)^{-1}(1 + h + a^2)^{-1}\right]^{\frac{1}{2}}\right\}^{-1} \tag{19}$$

In the CLL model (15), it is more difficult to give an exact meaning of the three parameters. However, if we think that h is a small parameter, as it is always assumed in our following calculations, the meaning of b and a is substantially the same as in the model with $h = 0$.

The procedure that we follow is that of applying the model to the calculation of some physical quantities; the comparison with experiments gives us information about the values to be attributed to the parameters. We have employed the CLL model to calculate drag, lift, and heat transfer coefficients in free-molecular flow.[33,34] Let us denote by T_0 the stagnation temperature, given for monatomic gases by $T_0 = T_\infty(1 + 2S^2/5)$, where the speed ratio $S = v_\infty/\sqrt{2RT_\infty}$ and T_∞ and v_∞ are the temperature and the bulk speed, respectively, of the incoming flow. Concerning C_D, the reported figures[33,34] for normal incidence show that the theoretical predictions, for high values of the ratio $r = T_w/T_0$, are higher than the experimental data. The same also holds for the results supplied by diffuse and specular reemission and therefore by the Schaaf theory based on the accommodation coefficient σ' and by the CL model as well.

In this connection, we recall that, as remarked by Knuth[3] and Liu et al.,[4] if the flow of normal momentum p_{nr} of the reemitted molecules is written, according to Schaaf, as $p_{nr} = (1 - \sigma_n)p_{ni} + \sigma_n p_{nw}$, from the well-known formulas for p_{ni} and p_{nw} (Ref. 2), then it follows that for any angle of attack θ, there exists a critical temperature ratio r_c, function of θ, for which $p_{ni} = p_{nw}$ and, then, $p_{nr} = p_{ni} = p_{nw}$, for any reemission model unless $\sigma_n \to \infty$. The total normal momentum flow turns out to be $p_n = 2p_{ni}$ for $r = r_c$, for any reemission model. Therefore, the values of C_D for normal incidence that we have considered previously[33,34] for values of r close to $r_c = 10/\pi$ should approach the value 4 and thus be higher than the experimental data of Bellomo et al.[19] This remark gives some justification of the predictions of our CL and CLL models for values of r close to r_c. Of course, we have the same situation also for different values of the angle of attack, but it is sufficient, considering normal incidence, to note this effect. Certainly, new experiments would be welcome to clarify the question, but at present it seems to us that it would be interesting to find a model, also if very schematic, able to predict values of C_D fitting the data of Legge et al.[19] in the neighborhood of r_c.

New Model for the Scattering Kernel

Mention is made that, in correspondence of high values of r, close to r_c, values of C_D lower than those provided by Maxwell's and CL models are provided by the elastodiffuse model.[35,36] This kernel has the following expression:

$$\mathcal{R}(\mathbf{c}' \to \mathbf{c}) = \frac{c_x \delta(c - c')}{\pi c^3} \tag{20}$$

It is easy to see that, according to Eq. (20), the reflected flow of tangential momentum $p_{\tau r}$ vanishes as in the case of diffused reemission, so that $\sigma_t = 1$. On the other hand, the reemitted and incident energy flows are equal ($E_r = E_i$), so that $\sigma_E = 0$ as in the case of specular reemission. In the case of extreme hypersonic flow ($T_\infty = 0$), we can simplify the expression of p_{nr} and make use of the formula $p_{nr} = \frac{4}{3}(\rho_\infty v_\infty^2/2)\sin\theta$, so that, for any value of the angle of attack and of the temperature ratio, we get $C_D = \frac{10}{3}$.

The elastodiffuse kernel was introduced only to give a mathematical example, but its capability to predict values of $C_D < 4$ is interesting.[31] At this point, the question is open whether it is possible to construct more realistic models that retain the characteristic property of giving, for normal incidence, values of C_D less than 4 for $r = 10/\pi$, and thus values of C_D closer to the experimental data[19] in the neighborhood of r_c. A possible idea that we exploit here is the approach already used in our previous papers[33,34] and described by Eq. (10) but starting from an expression of the tentative kernel $\mathcal{R}(\mathbf{c}' \to \mathbf{c})$ in which only the magnitudes of the velocity vectors appear in the argument of the exponential. We have tried

$$\mathcal{R}(\mathbf{c}' \to \mathbf{c}) = b\sqrt{a}\, c_x f_0(c) \exp\left[-a\beta_w(c - c')^2\right] \tag{21}$$

where a and b are constant parameters. The expression of this kernel for $a \to \infty$ tends to a distribution containing Dirac's delta:

$$\mathcal{R}(\mathbf{c}' \to \mathbf{c}) \to \mathcal{R}(\mathbf{c}' \to \mathbf{c}) = b(\pi/\beta_w)^{\frac{1}{2}} c_x f_0(c)\delta(c - c') \tag{22}$$

According to kernel (22) the magnitudes of the velocities of a molecule before and after the interaction with the wall are equal.

This idea was drawn from the elastodiffuse model given by Eq. (20). The kernel given by Eq. (21) mitigates this assumption because it contains a Gaussian function the width of which increases as the value of the parameter a decreases. The limiting model (22) contains just one parameter b, which is proportional to the fraction of molecules that preserve their kinetic energy in the interaction with a solid wall. The parameter a appearing in the full kernel (21) measures the extent of energy lost or gained in the interaction of the wall by the fraction of molecules that have a scattering that tends to preserve the kinetic energy. To be precise, this fraction modifies its speed with a statistical dispersion of the order of a multiple $a^{-1/2}$ of the molecules fully thermalized with the wall. When a becomes smaller and smaller, the molecules lose memory of their impinging kinetic energy and we approach pure diffusion. Both kernels give $p_{\tau r} = 0$ and thus $\sigma_t = 1$. For this purpose, we note that in some experiments by Liu et al.,[4] σ_t was found to be approximately unity; this result was attributed to backscattering. This value agrees with the values obtained by other authors cited in Ref. 4.

We first consider kernel (22), containing just one parameter b. In this case, Eqs. (11) and (12) give

$$H_L(c) = 2\sqrt{\pi}\, b c^3 \beta_w^{\frac{3}{2}} \exp(-\beta_w c^2)$$
$$I_L = 1 - 15\pi b\sqrt{2}/64 \tag{23}$$

[We note that the values of b must be chosen so that the restrictions $H_L(c) < 1$, $I_L > 0$ are satisfied.]

We have applied this new kernel to the calculation of drag, lift, and heat transfer coefficients, whose definitions are recalled in the Appendix. Figures 1 and 2 present some numerical results for C_D and C_H vs the temperature ratio T_w/T_0, for some values of the parameter b. In all figures of this paper, $*$, o and + denote three sets of experimental data[19] reported for helium impinging on gold at normal incidence. No information is given about the error bars,[19] but, subsequently, Legge et al.[37] specified that the accuracy of the heat transfer data is estimated to be 30% and the accuracy of the force measurements to be 10% near normal incidence and that even larger errors might occur for the tangential-force components. The uncertainty in the angle of attack is estimated to be 5 deg.

We first consider Fig. 1, showing the behavior of the drag coefficient C_D: The curves corresponding to different values of b cross at one point, but the experimental data are so scattered that it is impossible to conclude whether this feature might be considered acceptable or not. The curves in the region of high values of $r = T_w/T_0$ are noticeably closer to the experimental values than those obtained using the CL[19] or CLL[33,34] models. We stress that kernel (22) contains just one parameter but gives good results. For $b = 0.6$, the curve presents a slight inflection point.

We do not present results about the dependence of C_D on the angle of attack because the tangential momentum carried away by reflected molecules is zero and, moreover, C_{nr} is independent on the angle θ.

Also, the predictions of the model for reemitted intensity patterns in the case of an impinging monochromatic beam are trivial because $c_x^{-1} \mathcal{R}(\mathbf{c}' \to \mathbf{c})$ turns out to be a function only of the speeds. Therefore, the quantity

$$N(\theta, \phi) = \cos\theta \int_0^\infty \mathcal{R}(\mathbf{U} \to \mathbf{c}) c^3\, \mathrm{d}c \tag{24}$$

which gives this intensity,[7–9] turns out to be proportional to $\cos^2\theta$ as in the case of complete diffusion.

Figure 2 refers to the heat coefficient C_H. By increasing b, a better approximation of the experimental data is attained in the region of high values of r but worse in the region of low r. Moreover, the change of curvature in the curve corresponding to $b = 0.6$ is a bad, unexpected feature, and the model seems to be inappropriate for the study of heat transfer. It is also well known that, in the case of thermal diffusion, the recovery value r_0, of r at which the heat exchange vanishes, is 1.25; this value was observed in experiments on a hypersonic flow of a monatomic gas on rough technical surfaces. Recently, an increase in r_0 was found with different crystals[37]: The largest r_0 values were found for normal incidence; the values obtained according to model (24) are lower.

On the contrary, the CLL model, besides giving the right curvature of the heat transfer vs r (Refs. 33 and 34), provides results of r_0 higher than 1.25 as well. Consider, for instance, the case $h = 0.01$ and, for every value of a, the maximum value of b, according to Eq. (19): We have $r_0 = 1.287$ for $a = 0.7$, $r_0 = 1.333$ for $a = 0.8$, $r_0 = 1.442$ for $a = 0.9$ in agreement with some reported data.[37] The situation is similar for $h = 0.05$, as can be seen in Fig. 1 of our previous papers,[33,34] where, however, the values of r_0 are very close to 1.25. In general, the values of r_0 increase with a (we recall that this parameter is a measure of the fact that $\mathbf{c}$ is different from $\mathbf{c}'$).

At this point we investigate whether kernel (21), which generalizes the previous one and contains two parameters, gives an improvement of the results. In this case we get

$$H(c) = \sqrt{a}\, b(1 + a)^{-2} \exp(-u^2/a) \times \left[\exp(-u^2)(1 + u^2) + \sqrt{\pi}\, u\left(\tfrac{3}{2} + u^2\right)(1 + \operatorname{erf} u)\right] \tag{25}$$

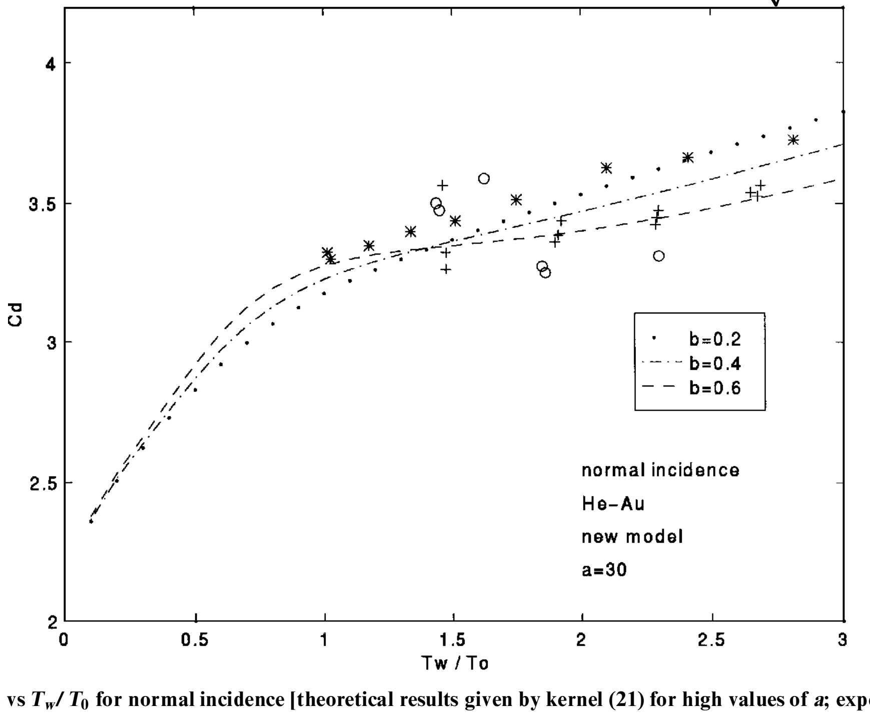

Fig. 1 Drag coefficient C_D vs T_w/T_0 for normal incidence [theoretical results given by kernel (21) for high values of a; experimental data for He on Au from Ref. 19]; $*$, o and + denote three sets of experimental data.

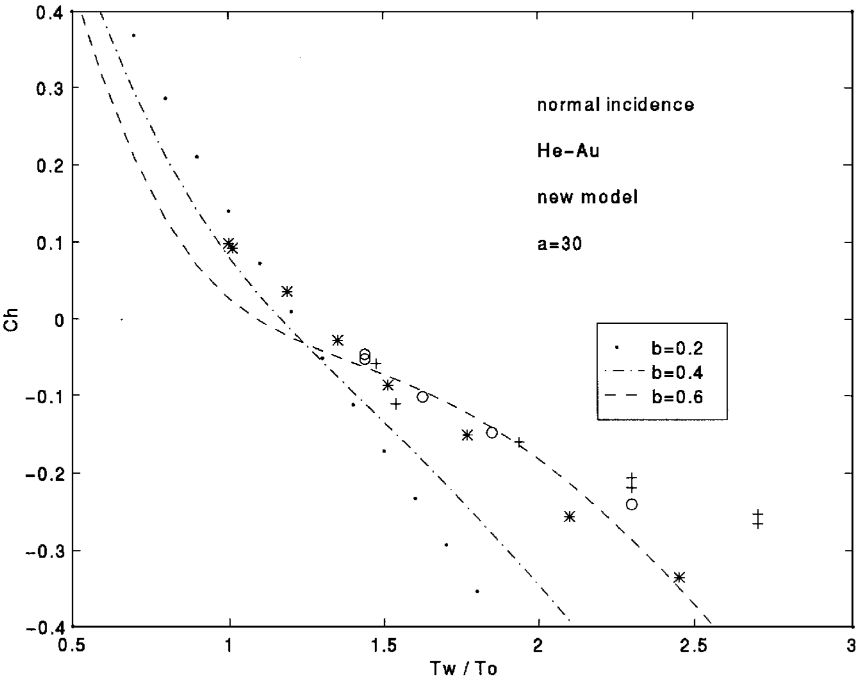

Fig. 2 Heat coefficient C_H vs T_w/T_0 for normal incidence [theoretical results given by kernel (21) for high values of a; experimental data for He on Au from Ref. 19]; *, σ and + denote three sets of experimental data.

where

$$u = a\sqrt{\beta_w}(1+a)^{-\frac{1}{2}}c \tag{26}$$

$$I = 1 - (I_1 + I_2 + I_3) \tag{27}$$

$$I_1 = 2b\sqrt{a}(1+a)^{-4}\left[\tfrac{1}{2} + a^2(1+a)^{-2}\right] \tag{28}$$

$$I_2 = \tfrac{3}{8}ba^{\frac{3}{2}}\pi(1+2a)^{-\frac{7}{2}}[3 + 6a + 5a^2] \tag{29}$$

$$I_3 = \frac{ba^{\frac{3}{2}}}{4(1+2a)^{\frac{5}{2}}}\left\{24G_4 + 48a^2(1+2a)^{-1}G_6\right\} \tag{30}$$

where

$$G_n = \int_0^{\psi}\cos^n\theta\,\mathrm{d}\theta, \qquad \psi = \arctan\frac{a}{\sqrt{1+2a}} \tag{31}$$

For each value of $a > 0$, we impose that b is limited by the restrictions $I > 0$, $H(c) < 1$. We remark that the present model, at variance with Maxwell's and the CL model, does not include specular reemission as a particular case.

Some calculations made by applying this new kernel to obtain the normal drag coefficient give the following results:

$$C_{Nr} = (2/v_\infty)(A_1 + A_2 + A_3) \tag{32}$$

$$A_1 = \frac{2b\sqrt{a}\exp(-U^2/a)}{3\sqrt{\beta_w}(1+a)^{\frac{5}{2}}}\left[U\exp(-U^2)\left(U^2 + \tfrac{5}{2}\right) + \sqrt{\pi}(1+\operatorname{erf}U)\left(\tfrac{3}{4} + 3U^2 + U^4\right)\right] \tag{33}$$

$$U = a\sqrt{\beta_w}(1+a)^{-\frac{1}{2}}v_\infty \tag{34}$$

$$A_2 = \frac{\sqrt{\pi}[1 - H(v_\infty)]}{2\sqrt{\beta_w}I} \tag{35}$$

$$A_3 = \frac{[1 - H(v_\infty)](A_{31} + A_{32} + A_{33})}{I} \tag{36}$$

$$A_{31} = -\frac{2b\sqrt{\pi}a^{\frac{3}{2}}\sqrt{(1+a)}}{\sqrt{\beta_w}(1+2a)^3}[1 + 2a^2(1+2a)^{-1}] \tag{37}$$

$$A_{32} = -\frac{b\sqrt{\pi a}}{2\sqrt{\beta_w}(1+a)^{\frac{9}{2}}}\left[1 + \frac{5a^2}{2(1+a)^2}\right] \tag{38}$$

$$A_{33} = -\frac{5b\sqrt{\pi(1+a)}a^{\frac{3}{2}}}{4\sqrt{\beta_w}(1+2a)^3}\left[3G_5 + 7a^2G_7(1+2a)^{-1}\right] \tag{39}$$

For the heat transfer coefficient, we obtain

$$C_{Hr} = (1/v_\infty^2)(B_1 + B_2 + B_3) \tag{40}$$

$$B_1 = \frac{b\sqrt{a}\exp(-U^2/a)}{\sqrt{\beta_w}(1+a)^3}\left[\exp(-U^2)\left(U^4 + \tfrac{9}{2}U^2 + 2\right) + \sqrt{\pi}(1+\operatorname{erf}U)\left(\tfrac{15}{4}U + 5U^3 + U^5\right)\right] \tag{41}$$

$$B_2 = \frac{2[1 - H(v_\infty)]}{\beta_w I} \tag{42}$$

$$B_3 = \frac{[1 - H(v_\infty)](B_{31} + B_{32} + B_{33})}{I} \tag{43}$$

$$B_{31} = -\frac{2b\sqrt{a}}{\beta_w(1+a)^5} \tag{44}$$

$$B_{32} = -\frac{15b\pi a^{\frac{3}{2}}(1+a)}{16\beta_w(1+2a)^{\frac{7}{2}}}[3 + 7a^2(1+2a)^{-1}] \tag{45}$$

$$B_{33} = -\frac{4ba^{\frac{3}{2}}(1+a)}{\beta_w(1+2a)^{\frac{7}{2}}}\left[\tfrac{9}{2}G_6 + 12a^2(1+2a)^{-1}G_8\right] \tag{46}$$

Of course, we have verified that the limits of these expressions of C_D and C_H for $a \to \infty$ are the same as we have already obtained directly, using kernel (22). Figures 3 and 4 show numerical results for C_D and C_H, respectively, vs the temperature ratio T_w/T_0, predicted by kernel (22) for $a = 5$ and some values of b. In Fig. 3, which shows the behavior of C_D, the inflection point is absent, so that the two-parameter kernel (23) seems to be an improvement with respect to the one-parameter kernel (24). The agreement with the experimental results is better for the highest values of b [in any case, b was assumed not to exceed 0.6 in order to satisfy the positivity condition of I and the inequality $H(c) < 1$]. Also, the results for

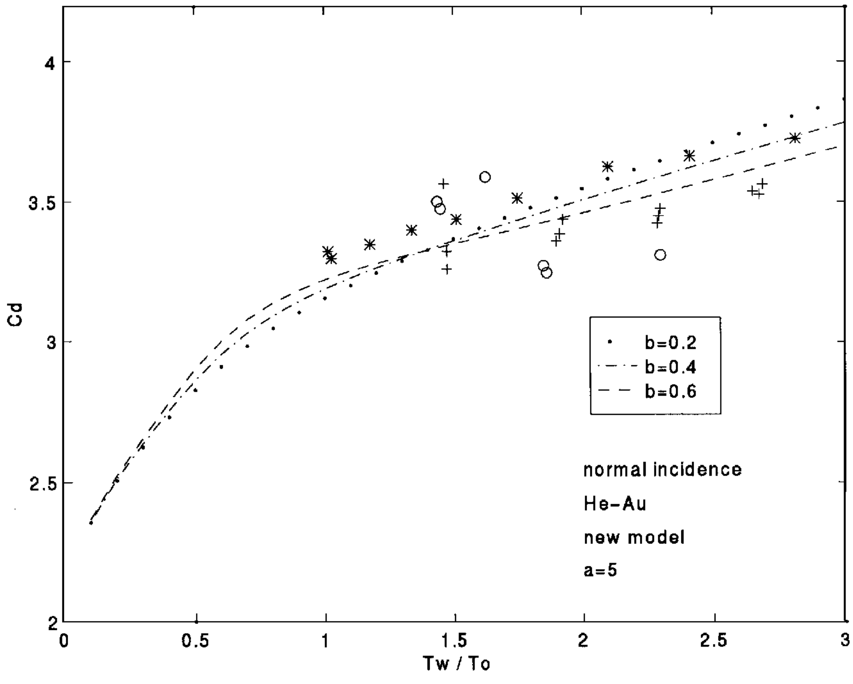

Fig. 3 Drag coefficient C_D vs T_w/T_0 for normal incidence [theoretical results given by kernel (21) for $a = 5$; experimental data for He on Au from Ref. 19]; *, σ and + denote three sets of experimental data.

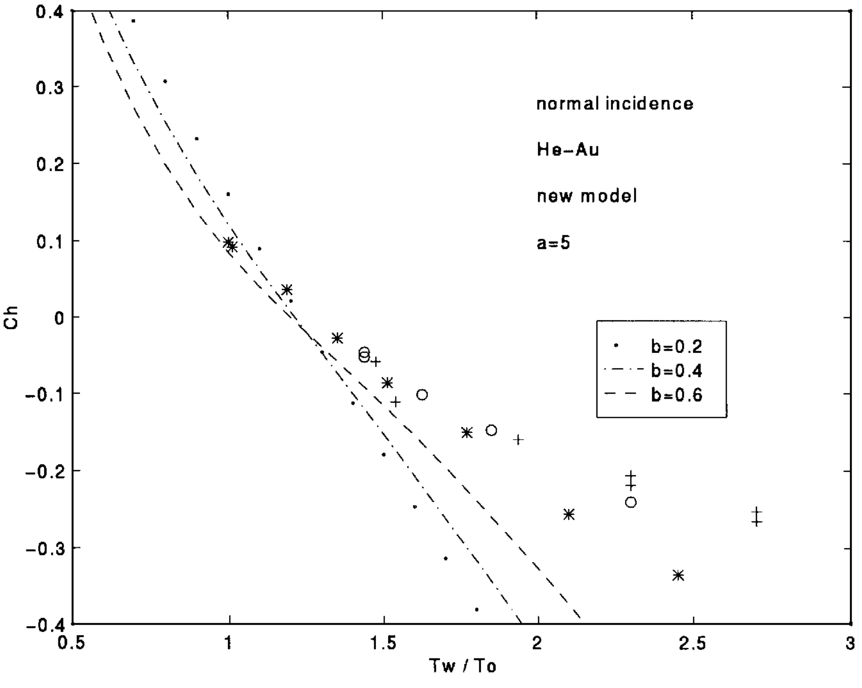

Fig. 4 Heat coefficient C_H vs T_w/T_0 for normal incidence [theoretical results given by kernel (21) for $a = 5$; experimental data for He on Au from Ref. 19]; *, σ and + denote three sets of experimental data.

C_H and for r_0, shown in Fig. 4, seem to be an improvement over those presented in Fig. 2. In particular, we remark that the values obtained for r_0 are close to 1.25. In any case, the results, also for the heat coefficient, are not worse than those given by thermal diffusion, which is recovered for $b = 0$.

Comparison with Experimental Data at Satellite Energies

It is important to study the drag coefficient for low values of T_0/T_w as well, as happens in the case of satellites circling the Earth at altitude within the upper atmosphere. Several results of laboratory experiments on forces due to free-molecular normal-momentum transfer at satellite energies are reported by Knuth[3] and Liu et al.[4] We compared the predictions given by our models with the experimental data for some monatomic gases reported in those papers.

We present results relative to $C_n = C_{ni} + C_{nr}$ vs the angle ϕ between the normal-to-the-surface and the impinging beam, measured in degrees, so that $\phi = 90 - \theta$, where θ is the angle of attack used in our previous papers. Because the gases considered in the present paper are monatomic and the incident beam is very narrow, the energy of the impinging beam is approximated by $E_i = \frac{1}{2}mv_\infty^2 = 5kT_0/2$. From the reported experimental values of α_{NM} and α'_{NM} (Refs. 3 and 4), it is easy to deduce $C_n = \alpha_{NM}(C_{ni} + C_{nw}) = \alpha'_{NM} C_{ni}$.

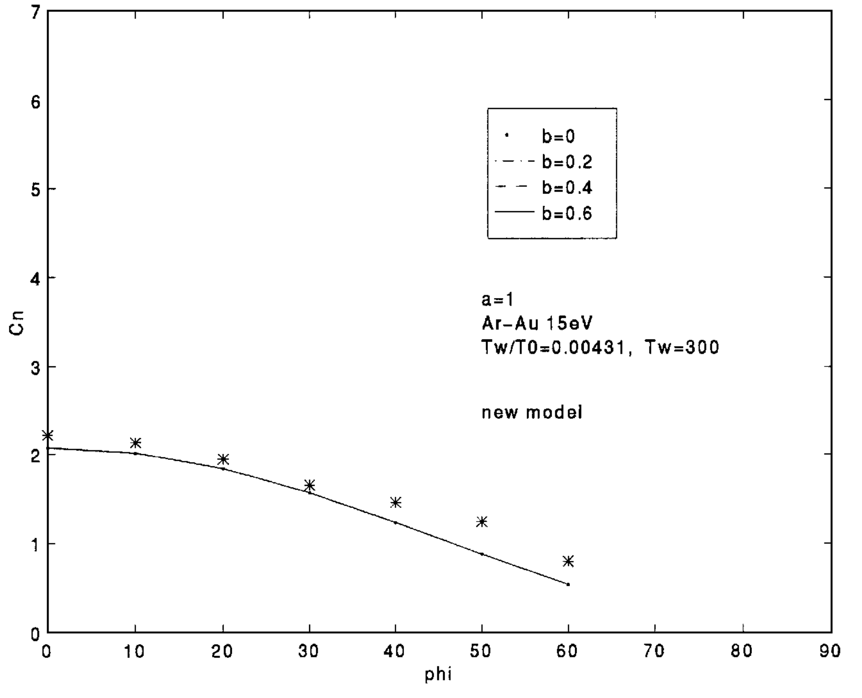

Fig. 5 Normal drag coefficient C_n vs the incidence angle ϕ [theoretical results given by kernel (21) for a monatomic gas impinging at 15 eV on a surface at T_w = 300–435 K; experimental data by Knechtel and Pitts[38] for Ar on Au].

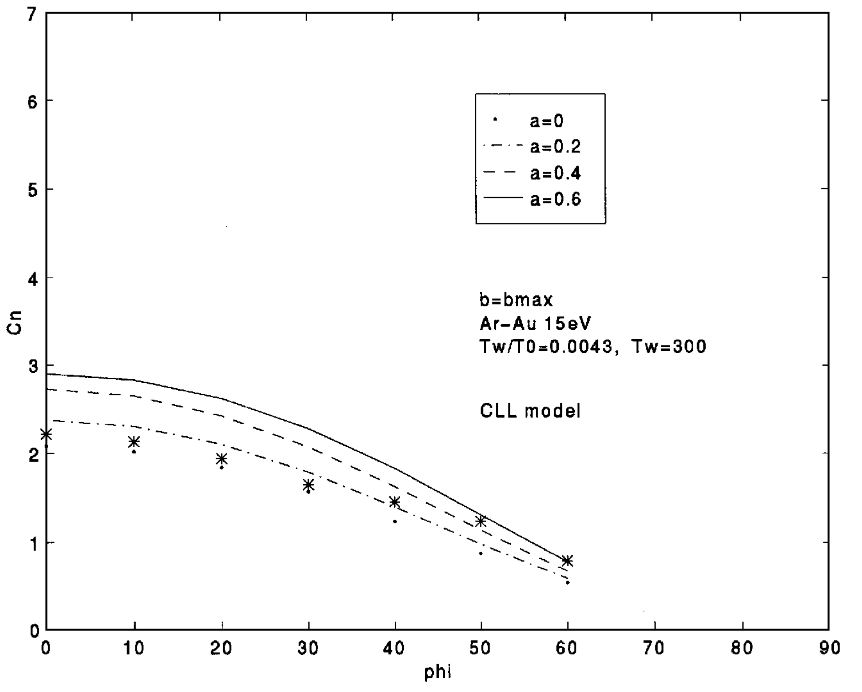

Fig. 6 Normal drag coefficient C_n vs the incidence angle ϕ (theoretical results given by the CLL model for a monatomic gas impinging at 15 eV on a surface at T_w = 300–435 K; experimental data by Knechtel and Pitts[38] for Ar on Au).

Figure 5 illustrates the results of kernel (21) for $E = 15$ eV and $T_w = 300$ K. Experimental data on the interaction of argon on vapor-deposited gold have been obtained by Knechtel and Pitts.[38] The agreement is rather good, but it is evident that the curves corresponding to different values of b coincide and are identical to the curve given by thermal diffusion ($b = 0$). In fact, for $T_w/T_0 \longrightarrow 0$, this kernel tends to the thermal diffusion kernel. Therefore, we do not present the other values that we have obtained in the case of small values of T_w/T_0.

We compared the experimental data with the CLL model, obtaining more interesting results. In our calculations, we always put $h = 0.001$, because h is a small parameter, introduced only for theoretical reasons, that can be maintained fixed in all cases. In practice, the effective parameters of the kernel are a and b. For every value of a, we have chosen $b = b_{\mathrm{Max}}$, according to formula (19). In all figures, the dots correspond to thermal diffusion and the asterisks denote the experimental data.

Figures 6 and 7 refer to the data by Knechtel and Pitts[38]; Fig. 6 regards argon impinging at 15 eV on background-contaminated vapor-deposited gold surfaces at 300–435 K. Figure 7 is the same for a surface of thin rolled aluminum sheets. The theoretical results do not change appreciably if we let $T_w = 300$ or 435 K. In both cases, the data are close to thermal diffusion ($a = 0$) for low values of ϕ, from 0 to 30 deg, but are better approximated by curves corresponding to $a = 0.2, 0.4$ for higher values of ϕ, from 40 to 60 deg. The same comment can be made about

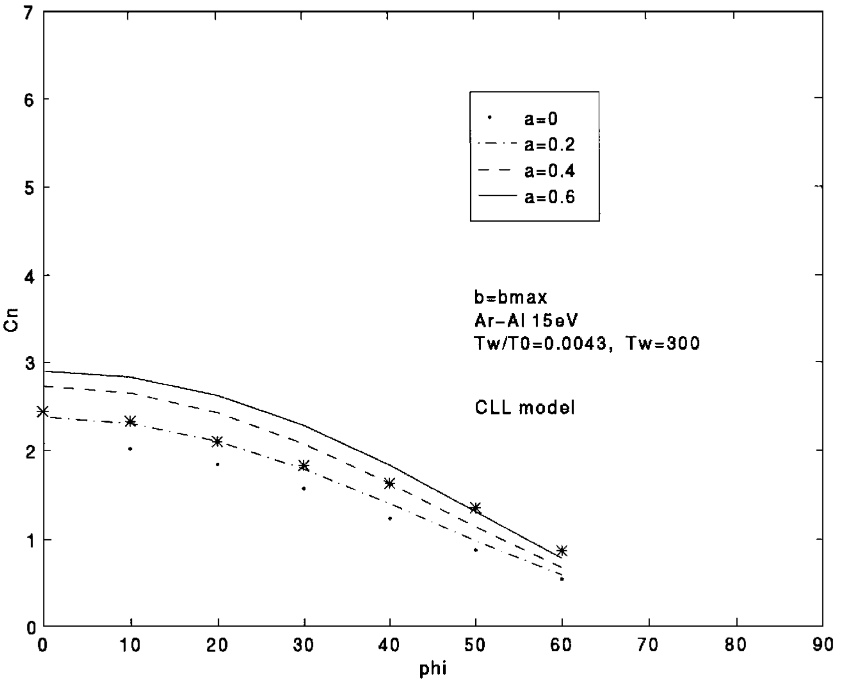

Fig. 7 **Normal drag coefficient C_n vs the incidence angle ϕ (theoretical results given by the CLL model for a monatomic gas impinging at 15 eV on a surface at T_w = 300–435 K; experimental data by Knechtel and Pitts[38] for Ar on Al).**

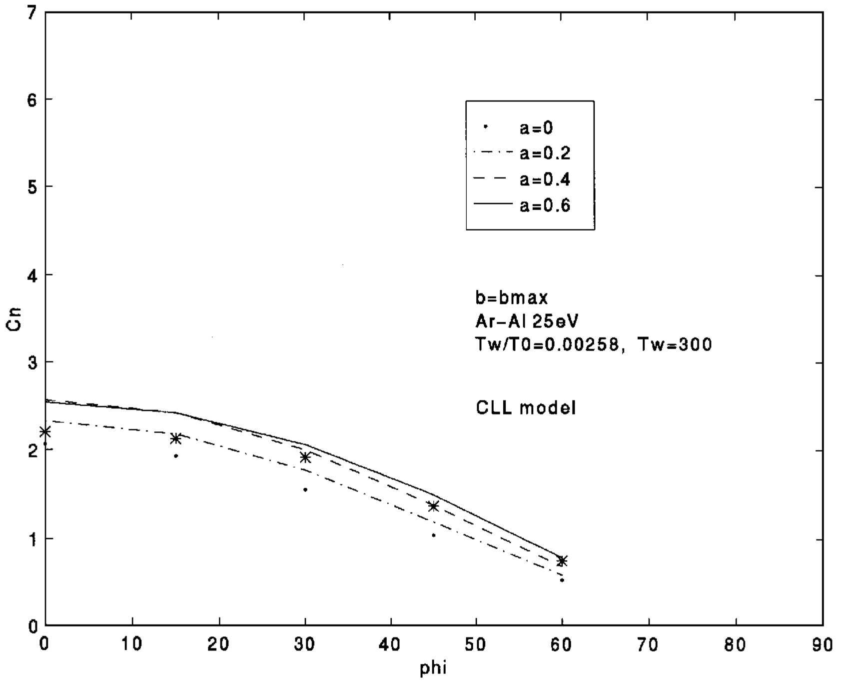

Fig. 8 **Normal drag coefficient C_n vs the incidence angle ϕ (theoretical results given by the CLL model for a monatomic gas impinging at 25 eV on a surface at T_w = 300 K; experimental data by Doughty and Schaetzle[39] for Ar on Al).**

Figs. 8 and 9, where the comparison of our theory is made with the data presented by Doughty and Schaetzel[39] on argon impinging at 25 eV on background-contamined aluminum and on fresh varnish at $T_w = 300$ K.

We have also considered the data of Liu et al.[4] on helium impinging at 1 eV on a 6061-T6 aluminum surface at 300 K. In this case, the results, shown in Fig. 10, are far from those given by thermal diffusion. We obtain the best agreement between theory and experiments for $a = 0.7, 0.8$.

Seidl and Steinheil[40] present results for helium at 0.05 eV on surfaces at $T_w = 300$ K. In this case, the energy of the impinging gas is not very high; our results in this case are practically the same given by thermal diffusion. The agreement is good for an uncleaned surface of gold, as shown in Fig. 11, whereas it is much less satisfactory for a mechanically polished sapphire surface (see Fig. 12). On the whole, the agreement of the CLL model with experiments is satisfactory.

Parameters of Models and Accommodation Coefficients

To relate the parameters of the kernels to some more usual parameters having a physical meaning, we recall the definition of the accommodation coefficients α_k introduced by Kuščer et al.,[36] who

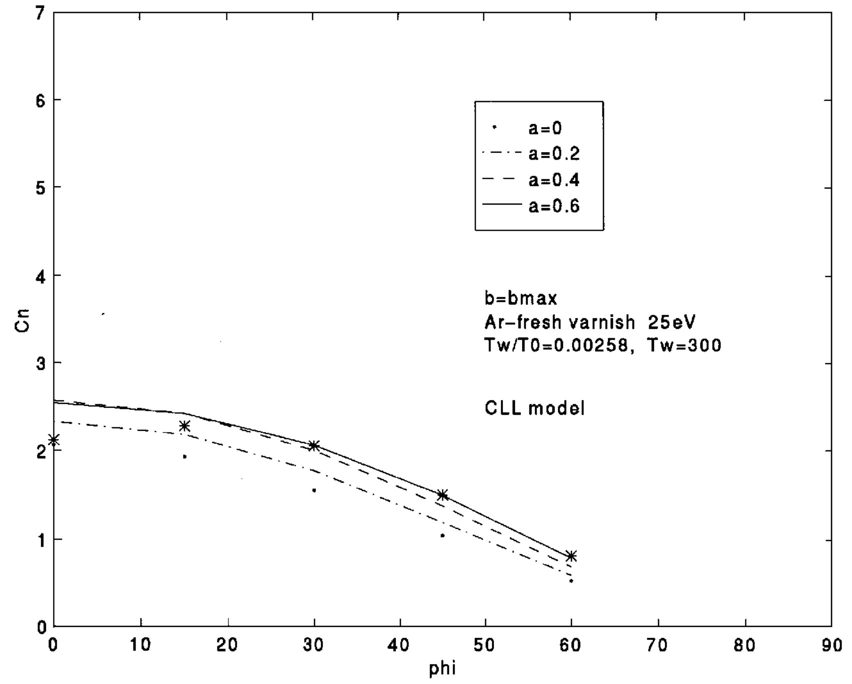

Fig. 9 Normal drag coefficient C_n vs the incidence angle ϕ (theoretical results given by the CLL model for a monatomic gas impinging at 25 eV on a surface at T_w = 300 K; experimental data by Doughty and Schaetzle[39] for Ar on fresh varnish).

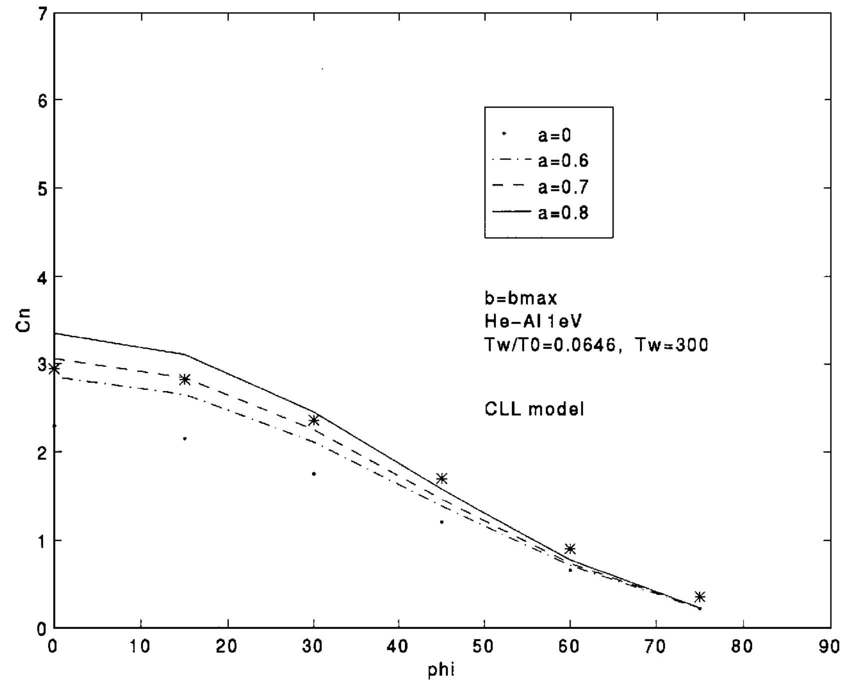

Fig. 10 Normal drag coefficient C_n vs the incidence angle ϕ (theoretical results given by the CLL model for a monatomic gas impinging at 1 eV on a surface at T_w = 300 K; experimental data by Liu et al.[4] for He on Al).

also evaluated these coefficients in the cases of the CL model and the elastodiffuse model:

$$\alpha_{ik} = \frac{(Q_i, Q_k)_B - (Q_i, \bar{Q}_k)_B}{(Q_i, Q_k)_B - (Q_i, N_k)_B} \tag{47a}$$

$$N_k = \frac{(Q_k, Q_0)_B}{(Q_0, Q_0)_B}, \qquad \bar{Q}_k = AQ_k$$

$$Q_0 = 1, \qquad Q_1 = |c_x|, \qquad Q_2 = c_y$$
$$Q_3 = c_z, \qquad Q_4 = c^2, \qquad Q_7 = |c_x| c^2 \tag{47b}$$

$$(g, h)_B = \int_{c_x > 0} c_x\, g(\mathbf{c})\, h(\mathbf{c})\, f_0\, \mathrm{d}\mathbf{c} \tag{48}$$

$$Ah = (c_x f_0)^{-1} \int_{c_x' < 0} R(\mathbf{c}' \to \mathbf{c})\, |c_x'|\, f_0'\, h'\, \mathrm{d}\mathbf{c}' \tag{49}$$

The basic parameters are α_{11}, α_{22}, and α_{44}, which are, respectively, accommodation coefficients for normal momentum, tangential momentum, and energy.

In principle, one might invert the expressions of the accommodation coefficients as functions of the parameters of the kernel, to obtain the parameters of the kernel as functions of the basic accommodation coefficients. The other accommodation coefficients

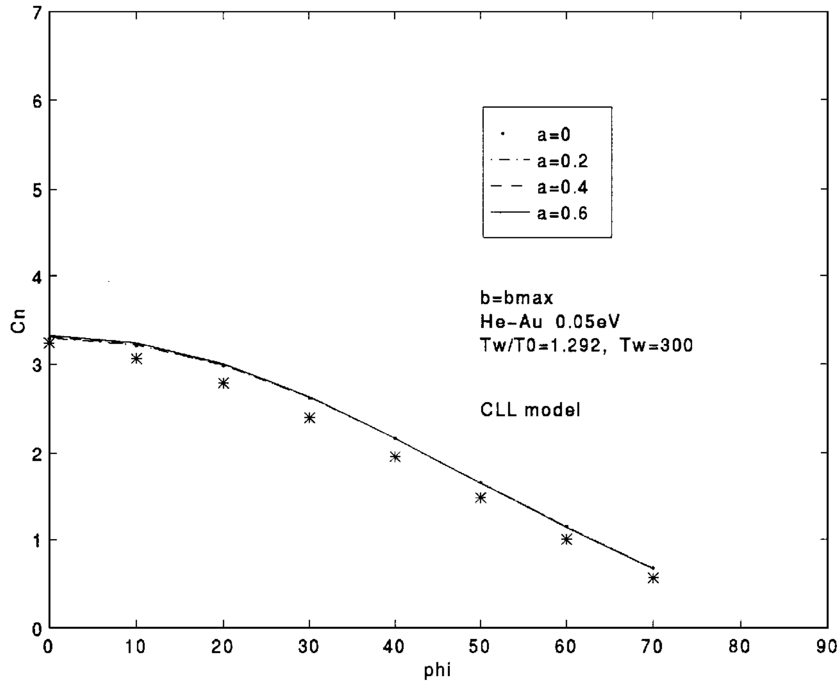

Fig. 11 Normal drag coefficient C_n vs the incidence angle ϕ (theoretical results given by the CLL model for a monatomic gas impinging at 0.05 eV on a surface at T_w = 300 K; experimental data by Seidl and Steinheil[40] for He on Au).

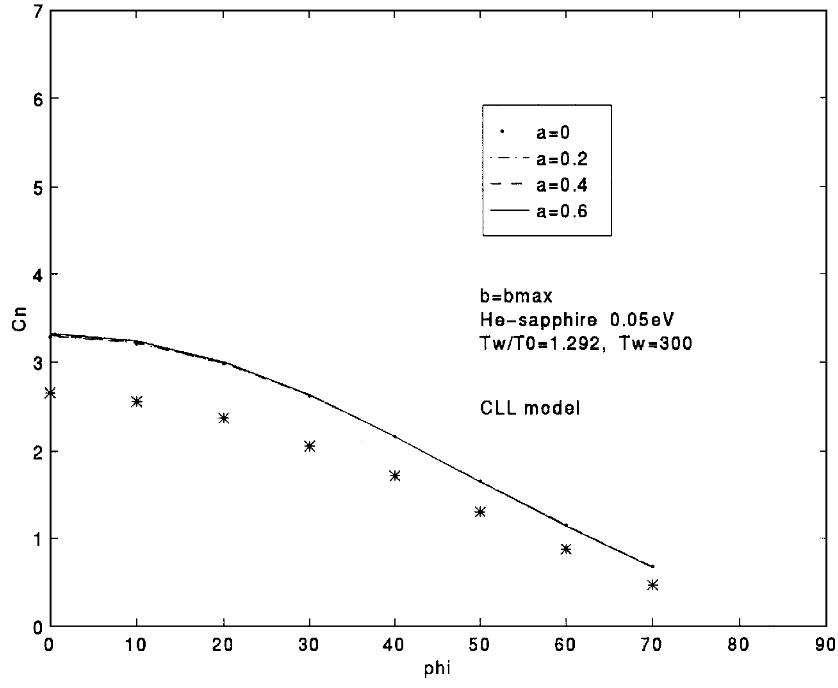

Fig. 12 Normal drag coefficient C_n vs the incidence angle ϕ (theoretical results given by the CLL model for a monatomic gas impinging at 0.05 eV on a surface at T_w = 300 K; experimental data by Seidl and Steinheil[40] for He on mechanically polished sapphire).

then can be thought of as functions of the basic ones. In the case of kernel (24), easy calculations yield

$$\alpha_{22} = 1, \qquad \alpha_{25} = 1 \tag{50a}$$

$$\alpha_{11} = (1 - \pi/4)^{-1}\left[1 - \frac{\pi}{4}\left(\frac{35}{24\sqrt{2}}b + \frac{(1-b)^2}{1 - 15\sqrt{2}\pi b/64}\right)\right] \tag{50b}$$

$$\alpha_{14} = 1 - 4(1-b)\left[\frac{1 - 135\sqrt{2}\pi b/512}{1 - 15\sqrt{2}\pi b/64} - 1\right] \tag{50c}$$

Using Eq. (50b), it is possible to relate the parameter b to the accommodation coefficient α_{11} and then, with similar calculations, to give the expressions of the other coefficients α_{ik}, for instance α_{14}, as functions of α_{11}.

In the case of kernel (21), this procedure, although clear in principle, is difficult in practice because the expressions of α_{ik} are cumbersome. A similar consideration could be repeated for the CLL model as well; in a recent paper, we gave the analytical expressions of the more usual coefficients α_{ik} for the CLL model as functions of the parameters of the model.[34] We recall, however, that the basic idea behind the collision kernel is to be able to represent the outcome of a very complicated physical process, the interaction of gas molecules with a solid wall. Hence, the parameters appearing in the models are not always immediately interpreted. For a long time, people were happy with the very rudimentary model proposed by Maxwell more than a century ago. The model proposed by Nocilla[5] has sufficient flexibility—and as such it was first exploited by Hurlbut and Sherman[6]—but has the disadvantage of not being amenable to agreement with the properties of scattering kernels. The other models known so far, including those previously presented by the authors,[13,34] share with Maxwell's model the peculiar feature of having an accommodation coefficient for normal momentum, as defined by Schaaf,[2] that remains finite. Now this coefficient, as we have stressed before, is not necessarily finite in all circumstances, as has been remarked.[3,4] This fact came dramatically in the foreground with the experiments of Bellomo et al.,[19] which seem to occur exactly in the range of parameters in which the Schaaf coefficient may tend to infinity. The only scattering kernel available that avoided the difficulty of having a possibly unbounded normal momentum coefficient, in the Schaaf sense, was the elastodiffuse kernel.[32] This model is clearly schematic and not sufficiently flexible to represent all possible experimental situations. Thus, we propose the new model (21) that embodies some features of the elastodiffuse kernel and of thermal diffusion.

Of course, physical parameters such as the ratio of the gas molecular mass to the mass of the wall atoms do not enter our model or any of the other available scattering kernels. Rather, the parameters of these kernels are the measure of certain tendencies in the physical process occurring during the gas–surface interaction. We clearly are experimenting with kernels and thus do not pretend that our parameters have a direct physical significance, but we can give them a qualitative interpretation.

Concluding Remarks

We introduce a new model for gas–surface interaction that produces results in agreement with the experimental data for the drag of a flat plate in hypersonic free-molecular flow. All of the previous models were unable to predict these results. For low values of T_w/T_0, the kernel gives practically the same results as the thermal diffusion model.

On the contrary, the CLL model gives good results for the drag at small values of T_w/T_0, corresponding to the conditions of satellites circling the Earth at altitude within the upper atmosphere. Moreover, the CLL models with respect to the new one also give better results for the heat transfer coefficient C_H in the interval of the temperature ratio.

The agreement obtained here does not close the question but rather opens a new chapter in the study of gas–surface interaction modeling, a subject that has been neglected for many years. It also calls for more detailed and accurate experimental data.

Appendix: Drag, Lift, and Heat Transfer Coefficients

We calculate the coefficients of drag, lift, and heat transfer in free-molecular flow. As usual, we assume as incident distribution function

$$f_\infty = (2\pi R T_\infty)^{-\frac{3}{2}} n_\infty \exp[-\beta_\infty(\mathbf{c} - \mathbf{v}_\infty)^2] \tag{A1}$$

where n_∞ is the number density and $\beta_\infty = m/(2kT_\infty)$. Let us denote f^- and f^+ as the restrictions of $f(\mathbf{c})$ to negative and positive values of c_x, respectively, and f' denotes $f(\mathbf{c}')$.

The incident and reemitted fluxes Φ_i and Φ_r of a quantity ϕ are given by

$$\Phi_i = \int_{c'_x<0} |c'_x| \phi' f'^-_\infty \, d\mathbf{c}' = \int_{c_x>0} c_x P_n \phi P_n f^-_\infty \, d\mathbf{c} \tag{A2}$$

$$\Phi_r = \int_{c_x>0} c_x \phi f^+_\infty \, d\mathbf{c} = \int_{c_x>0} \phi \, d\mathbf{c} \int_{c'_x<0} |c'_x| \mathcal{R}(\mathbf{c}' \to \mathbf{c}) f'^-_\infty \, d\mathbf{c}'$$

$$= \int_{c'_x<0} |c'_x| f'^-_\infty \, d\mathbf{c}' \int_{c_x>0} \phi \mathcal{R}(\mathbf{c}' \to \mathbf{c}) \, d\mathbf{c} \tag{A3}$$

and $P_n(h) = h(-c_x)$. Then we define the coefficients

$$C_n = \frac{\Phi(c_x)}{(\rho v_\infty^2/2)} = \frac{[\Phi_i(c_x) + \Phi_r(c_x)]}{(\rho v_\infty^2/2)} = C_{ni} + C_{nr} \tag{A4}$$

$$C_\tau = \frac{\Phi(c_y)}{(\rho v_\infty^2/2)} = \frac{[\Phi_i(c_y) - \Phi_r(c_y)]}{(\rho v_\infty^2/2)} = C_{\tau i} - C_{\tau r} \tag{A5}$$

$$C_H = \frac{\Phi(c^2/2)}{(\rho v_\infty^2/2)} = \frac{[\Phi_i(c^2/2) - \Phi_r(c^2/2)]}{(\rho v_\infty^2/2)} = C_{Hi} - C_{Hr} \tag{A6}$$

where the expressions of the incident quantities C_{ni}, $C_{\tau i}$, C_{Hi} for a given angle of attack θ are well known.

In Refs. 33 and 34, the drag and lift coefficients are referred to the frontal area as in Ref. 19:

$$C_D = \frac{C_n \sin\theta + C_\tau \cos\theta}{\sin\theta} \tag{A7}$$

$$C_L = \frac{C_N \cos\theta - C_\tau \sin\theta}{\sin\theta} \tag{A8}$$

Acknowledgment

This work has been performed in the frame of the activity of Gruppo Nazionale per la Fisica Matematica of Consiglio Nazionale delle Ricerche and supported by Ministero dell'Università e della Ricerca Scientifica e Tecnologica.

References

[1] Maxwell, J. C., "On Stresses in Rarified Gases Arising from Inequalities of Temperature," *Philosophical Transactions of the Royal Society of London*, Vol. 170, No. 1, Appendix, 1879, pp. 231–256.

[2] Schaaf, S. A., "Mechanics of Rarefied Gases," *Handbuch der Physik*, Vol. 8/2, edited by S. Flugge, Springer, Berlin, 1963, pp. 591–624.

[3] Knuth, E. L., "Free-Molecule Normal-Momentum Transfer at Satellite Surfaces," *AIAA Journal*, Vol. 18, No. 5, 1980, pp. 602–605.

[4] Liu, S. M., Sharma, P. K., and Knuth, E. L., "Satellite Drag Coefficients Calculated from Measured Spatial and Energy Distributions of Reflected Helium Atoms," *AIAA Journal*, Vol. 17, No. 12, 1979, pp. 1314–1319.

[5] Nocilla, S., "The Surface Re-Emission Law in Free Molecule Flow," *Rarefied Gas Dynamics*, edited by J. A. Laurmann, Academic, New York, 1963, pp. 327–346.

[6] Hurlbut, F., and Sherman, F. S., "Application of the Nocilla Wall Reflection Model to Free-Molecule Kinetic Theory," *Physics of Fluids*, Vol. 11, March 1968, pp. 486–496.

[7] Cercignani, C., *Mathematical Methods in Kinetic Theory*, 2nd ed., Plenum, New York, 1990, pp. 52–56.

[8] Cercignani, C., *Theory and Application of the Boltzmann Equation*, Scottish Academic, Edinburgh, Scotland, UK, 1975, Chap. 3.

[9] Cercignani, C., *The Boltzmann Equation and Its Applications*, Springer–Verlag, New York, 1987, Chap. 3.

[10] Kuščer, I., "Reciprocity in Scattering of Gas Molecules by Surfaces," *Surface Science*, Vol. 25, No. 2, 1971, pp. 225–237.

[11] Cabrera, N., Celli, V., and Manson, J. R., "Theory of surface scattering and Detection of Surface Phonons," *Physical Review Letters*, Vol. 22, No. 8, 1969, pp. 346–348.

[12] Cabrera, N., Celli, V., Goodman, F. O., and Manson, J. R., "Scattering of Atoms by Solid Surfaces," *Surface Science*, Vol. 19, No. 1, 1970, pp. 67–92.

[13] Cercignani, C., and Lampis, M., "Kinetic Models for Gas-Surface Interaction," *Transport Theory and Statistical Physics*, Vol. 1, No. 2, 1971, pp. 101–114.

[14] Cercignani, C., "Scattering Kernels for Gas-Surface Interactions," *Transport Theory and Statistical Physics*, Vol. 1, No. 1, 1972, pp. 27–53.

[15] Cercignani, C., "Models for Gas-Surface Interaction: Comparison Between Theory and Experiment," *Rarefied Gas Dynamics*, edited by D. Dini, S. Nocilla, and C. Cercignani, Vol. 1, Editrice Tecnico Scientifica, Pisa, Italy, 1974, pp. 75–95.

[16] Cercignani, C., and Lampis, M., "Free Molecular Flow Past a Flat Plate in the Presence of a Nontrivial Gas-Surface Interaction," *Zeitschrift für Angewandte Mathematik und Physik*, Vol. 23, No. 5, 1972, pp. 713–728.

[17] Lampis, M., "Some Applications of a Model for Gas-Surface Interaction," *Rarefied Gas Dynamics*, edited by K. Karamcheti, Academic, New York, 1974, pp. 369–380.

[18] Cercignani, C., and Lampis M., "On the Recovery Factor in Free-Molecular Flow," *Zeitschrift für Angewandte Mathematik und Physik*, Vol. 27, No. 6, 1976, pp. 733–738.

[19] Bellomo, N., Dankert, C., Legge, H., and Monaco, R., "Drag, Heat Flux, and Recovery Factor Measurements in Free Molecular Hypersonic Flow and Gas-Surface Interaction Analysis," *Rarefied Gas Dynamics*, edited by O. M. Belotserkovskii, M. N. Kogan, S. S. Kutateladze, and A. K. Rebrov, Vol. 1, Plenum, New York, 1985, pp. 421–430.

[20] Cercignani, C., and Frezzotti, A., "Numerical Simulation of Supersonic Rarefied Gas Flows Past a Flat Plate: Effects of the Gas-Surface Interaction Model on the Flowfield," *Rarefied Gas Dynamics: Physical Phenomena*, edited by E. P. Muntz, D. P. Weaver, and D. H. Campbell, Vol. 118, Progress in Aeronautics and Astronautics, AIAA, Washington, DC, 1989, pp. 552–566.

[21] Karr, G. R., Gregory, J. C., and Peters P. N., "Free Molecule Drag and Lift Deduced from Shuttle Flight Experiment," *Rarefied Gas Dynamics*, edited by V. Boffi and C. Cercignani, Vol. 1, Teubner, Stuttgart, Germany, 1986, pp. 609–617.

[22] Hurlbut, F. C., "Particle Surface Interaction in the Orbital Context: A Survey," *Rarefied Gas Dynamics: Space-Related Studies*, edited by E. P. Muntz, D. P. Weaver, and D. H. Campbell, Vol. 116, Progress in Aeronautics and Astronautics, AIAA, Washington, DC, 1989, pp. 419–450.

[23] Hurlbut, F., "Two Contrasting Modes for the Description of Wall-Gas Interactions," *Rarefied Gas Dynamics: Experimental Techniques and Physical Systems*, edited by B. D. Shizgal and D. P. Weaver, Vol. 158, Progress in Astronautics and Aeronautics, AIAA, Washington, DC, 1994, pp. 494–506.

[24] Gaposchkin, E. M., "Calculation of Satellite Drag Coefficients," Lincoln Lab., TR 998, Massachusetts Inst. of Technology, Cambridge, MA, 1994.

[25] Collins, F. G., and Knox, E. C., "Parameters of Nocilla Gas/Surface Interaction Model from Measured Accommodation Coefficients," *AIAA Journal*, Vol. 32, No. 4, 1994, pp. 765–773.

[26] Koppenwallner, G., Johannsmeier, D., Klinkrad, H., Ivanov, M., and Kashkowski, A., "A Rarefied Aerodynamics Modelling System for Earth Satellites (RAMSES)," *Rarefied Gas Dynamics*, edited by J. Harvey and G. Lord, Vol. 2, Oxford Univ. Press, Oxford, England, UK, 1995, pp. 1366–1372.

[27] Hurlbut, F. C., "Gas Surface Interactions: Recent Observations and Interpretations," *Proceedings of the XXth Rarefied Gas Dynamics Symposium*, edited by Shen Ching, Beijing Univ. Press, Beijing, PRC (to be published) (Paper INV2).

[28] Cercignani, C., and Lampis, M., "Variational Calculation of the Slip Coefficient and the Temperature Jump for Arbitrary Gas-Surface Interactions," *Rarefied Gas Dynamics: Space-Related Studies*, edited by E. P. Muntz, D. P. Weaver, and D. H. Campbell, Vol. 116, Progress in Aeronautics and Astronautics, AIAA, Washington, DC, 1989, pp. 553–561.

[29] Cercignani, C., Lampis, M., and Lentati, A., "Some New Results About the Temperature Jump for a Mixture," *Proceedings of the VIIth European Conference on Mathematics in Industry*, edited by A. Fasano and M. Primicerio, Teubner, Stuttgart, Germany, 1994.

[30] Cercignani, C., Lampis, M., and Lentati, A., "Calculation of the Slip Coefficient for a Mixture," *Rarefied Gas Dynamics: Theory and Simulations*, edited by B. D. Shizgal and D. P. Weaver, Vol. 159, Progress in Astronautics and Aeronautics, AIAA, Washington, DC, 1994, pp. 565–573.

[31] Cercignani, C., Lampis, M., and Lentati, A., "Variational Calculation of the Temperature Jump for Polyatomic Gas," *Rarefied Gas Dynamics*, edited by J. Harvey and G. Lord, Vol. 2, Oxford Univ. Press, Oxford, England, UK, 1995, pp. 1023–1029.

[32] Cercignani, C., "Scattering Kernels for Gas-Surface Interaction," *Proceedings of the Workshop on Hypersonic Flows for Reentry Problems* (Antibes, France), Vol. 1, Institut National de Recherche en Information et en Automatique, Sophia Antipolis, France, 1990, pp. 9–29.

[33] Cercignani, C., Lampis, M., and Lentati, A., "On the Drag and Heat Transfer Coefficients in Free-Molecular Flow," *Rarefied Gas Dynamics*, edited by J. Harvey and G. Lord, Vol. 2, Oxford Univ. Press, Oxford, England, UK, 1995, pp. 1190–1196.

[34] Cercignani, C., Lampis, M., and Lentati, A., "A New Scattering Kernel in Kinetic Theory of Gases," *Transport Theory and Statistical Physics*, Vol. 24, No. 9, 1995, pp. 1319–1336.

[35] Legge, H., "Heat Transfer and Forces on a LiF Single-Crystal Disc in Hypersonic Flow," DLR, German Aerospace Research Establishment, Rept. DLR-IB 222-92 A 14, Göttingen, Germany, Aug. 1992.

[36] Klinc, T., and Kuščer, I., "Slip Coefficients for General Gas-Surface Interactions," *Physics of Fluids*, Vol. 15, No. 6, 1972, pp. 1018–1022.

[37] Legge, H., Toennies, J. P., and Lüdecke, J., "Measurements of Recovery Temperatures, Heat Transfer, and Force Accommodation Coefficients for a He Free Jet on a Clean Single Crystal of LiF (001)," *Rarefied Gas Dynamics*, edited by J. Harvey and G. Lord, Vol. 2, Oxford Univ. Press, Oxford, England, UK, 1995, pp. 998–994.

[38] Knechtel, E. E., and Pitts, W. C., "Experimental Momentum Accommodation on Metal Surfaces of Ions Near and Above Earth-Satellite Speed," *Rarefied Gas Dynamics*, edited by L. Trilling and H. Y. Wachman, Vol. 2, Academic, New York, 1969, pp. 1257–1266.

[39] Doughty, R. O., and Schaetzel, W. J., "Experimental Determination of Momentum Accommodation Coefficients at Velocities up to and Exceeding Earth Escape Velocity," *Rarefied Gas Dynamics*, edited by L. Trilling and H. Y. Wachman, Vol. 2, Academic, New York, 1969, pp. 1035–1054.

[40] Seidl, M., and Steinheil, E., "Measurement of Momentum Accommodation Coefficients on Surfaces Characterized by Auger Specroscopy, SIMS and LEED," *Rarefied Gas Dynamics*, edited by M. Becker and M. Fiebig, Vol. 2, DFVLR Press, Proz–Wahn, Germany, 1974, Paper E-9.

W. Oberkampf
Associate Editor